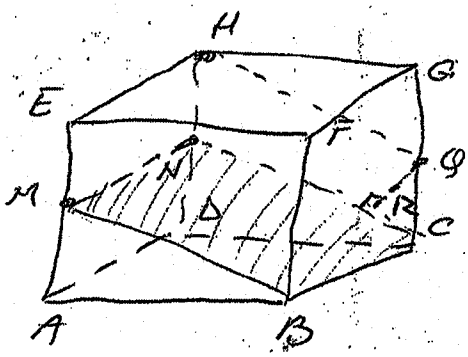
EN / 2025 / MODEL / III 6

$ABCDEFGH$ cub, $AB = 6$ cm

M, N mijloacele lui AE, DH

a) arată $V_{ABCDEFGH} = 216\text{ cm}^3$

b) calc. dist$(H, (CMN))$



---

a) $V = l^3 = 6^3\text{ cm}^3 = 216\text{ cm}^3$

b) (CMN): $C \in (CMN)$, $CB \parallel MN$, $MN \subset (CMN) \Rightarrow$

$\Rightarrow MN \subset (CMN) \Rightarrow (CMN) = (BCN)$

Folosim $d \parallel \alpha$, $\forall M \in d \Rightarrow$ dist(M, α) aceeași

Fie $HQ \parallel NC$, $Q \in CG \Rightarrow HQ \parallel (CMN)$ și $HNCQ$ paral

$(M \in HQ \parallel NC, HN \parallel QC) \Rightarrow HN = QC \Rightarrow QC = 3$ cm

Fie $QR \perp NC$, căutăm o altă dreaptă din (CMN) pt care $QR \perp d$

Cum $MN \perp (HDC)$ deoarece $MN \perp HD$

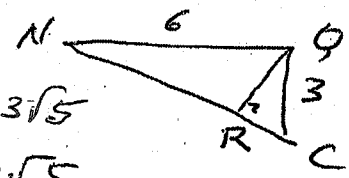
$MN \parallel AA_1$, $AA_1 \perp DC \Rightarrow MN \perp DC$

și $QR \subset (HDC) \Rightarrow MN \perp QR \Rightarrow QR \perp MN$

$\Rightarrow QR \perp (MNC) \Rightarrow QR$ e distanța cerută

$ND \parallel QC$, $ND = QC$, $N\hat{D}C = 90° \Rightarrow NDCQ$ dreptunghi $\Rightarrow$

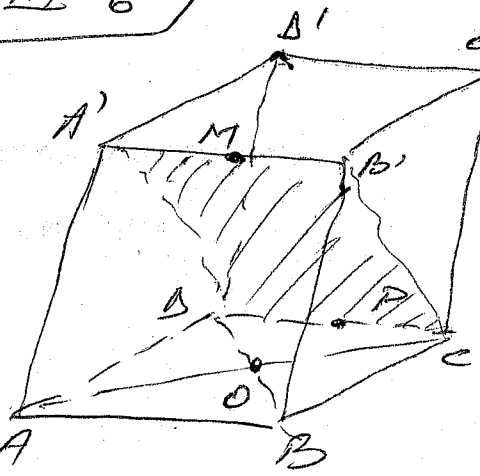
$m(N\hat{Q}C) = 90°$



T Pit $\Rightarrow NC = \sqrt{36+9} = \sqrt{45} \Rightarrow NC = 3\sqrt{5}$

$\Rightarrow QR = \frac{NQ \cdot QC}{NC} = \frac{6 \cdot 3}{3\sqrt{5}} = \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$ cm

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$ABCDA'B'C'D'$ cub cu $AB = 8$ cm

a) $V_{ABCDA'B'C'D'} = ?$

b) distanța de la O la (CMD) unde M e mijlocul lui $A'B'$

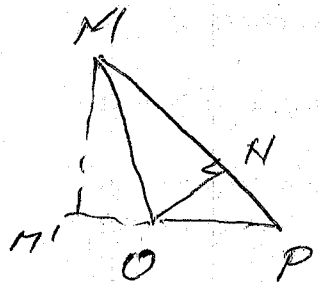
a) $V_{ABCDA'B'C'D'} = l^3 = 8^3 = 512\ cm^3$

b) $\left.\begin{array}{l} CD \subset (CMD) \\ M \in (CMD) \\ M \in A'B' \\ A'B' \parallel CD \end{array}\right\} \Rightarrow A'B' \subset (CMD) \Rightarrow (CMD) = (A'DC)$

Fie $ON \perp (A'DC)$, $NP \perp DC$, $NP \subset (A'DC)$, $DC \subset (A'DC)$ $\Rightarrow OP \perp DC$

Dar ΔDOC isoscel $\Rightarrow$

$\Rightarrow P$ mijl lui DC



$\left.\begin{array}{l} A_{OMP} = \frac{MP \cdot ON}{2} \\ A_{OMP} = \frac{OP \cdot MM'}{2} \end{array}\right\} \Rightarrow$

$\boxed{MP \cdot ON = OP \cdot MM'}$

M mijl $A'B'$, P mijl $DC \Rightarrow MPCB'$ paral $\Rightarrow MP = B'C = 8\sqrt{2}$

$\Rightarrow \boxed{MP = 8\sqrt{2}\ cm}$

$OP = \frac{BC}{2} = 4 \Rightarrow \boxed{OP = 4\ cm}$

$MM' \perp OP \Rightarrow M' \in AB$, M' mijl $AB \Rightarrow MM' = AA' = 8 \Rightarrow \boxed{MM' = 8\ cm}$

Înlocuim în $MP \cdot ON = OP \cdot MM'$

$\Rightarrow 8\sqrt{2} \cdot ON = 4 \cdot 8 \Rightarrow ON = \frac{32}{8\sqrt{2}} = \frac{4}{\sqrt{2}}$

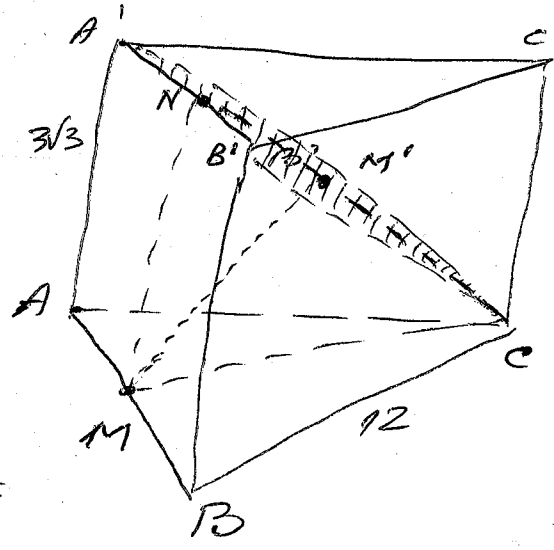
$\Rightarrow ON = \frac{4\sqrt{2}}{2} = 2\sqrt{2}\ cm$

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$ABCA'B'C'$ prismă dreaptă, $\triangle ABC$ echilateral, $AB = 12$, $AA' = 3\sqrt{3}$, M mijlocul lui AB

a) M care $A_l = 108\sqrt{3}$

b) distanța de la M la $(A'B'C)$



---

a) $A_l = 3 \cdot A_{ABB'A'} = 3 \cdot 12 \cdot 3\sqrt{3} =$

$= 108\sqrt{3}$

b) $A'C \equiv B'C$ (diagonale în dreptunghiuri congruente)

N fie $CN \perp A'B' \Rightarrow N$ mijl lui $A'B'$

M mijl AB $\Big\} \Rightarrow MN \parallel AA'$

$\Rightarrow \boxed{MN \perp A'B'}$

Fie $MM' \perp (A'B'C)$

$MN \perp A'B'$

$A'B'C(A'B'C)$ $\Big\} \xrightarrow[\text{Tales } 3\perp]{R_1}$ $M'N \perp A'B'$ Dar, $CN \perp A'B'$ $\Big\} \Rightarrow$

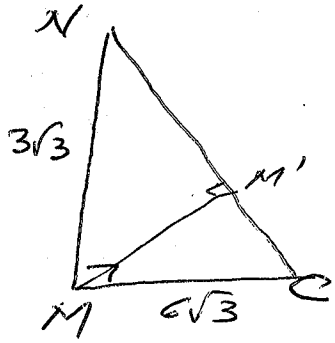
MC înălț. $\triangle$ echilateral $\Rightarrow M' \in CN$

$\Rightarrow CM = \frac{l\sqrt{3}}{2} = \frac{12\sqrt{3}}{2} = 6\sqrt{3}$

$NM \perp AB$

$NMC(ABB')$ $\Big\} \Rightarrow NM \perp (ABC)$

$(ABB') \perp (ABC)$ $\Rightarrow NM \perp MC$



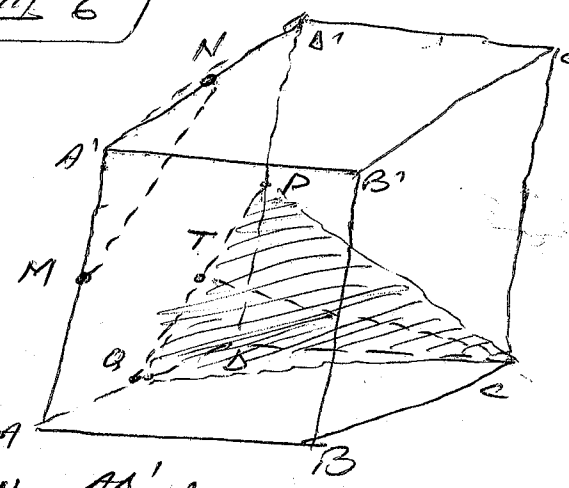
T. Pitag $\triangle NMC \Rightarrow NC^2 = NM^2 + MC^2 \Rightarrow NC^2 = (3\sqrt{3})^2 + (6\sqrt{3})^2$

$\Rightarrow NC^2 = 27 + 108 = 135 \Rightarrow NC = \sqrt{9 \cdot 15} = 3\sqrt{15}$

$MM' \perp NC \Rightarrow MM' = \frac{c_1 \cdot c_2}{i} = \frac{3\sqrt{3} \cdot 6\sqrt{3}}{3\sqrt{15}} = \frac{18\sqrt{15}}{15} = \frac{6\sqrt{15}}{5}$

$\Rightarrow MM' = \frac{6\sqrt{15}}{5}$

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$ABCDA'B'C'D'$ cub și M, N, P, Q mijloace

a) arată $MN = PQ$

b) dacă T mijloc al PQ, să se arate $CT \parallel (MNB)$

---

a) MN l mijl $\triangle AA'D' \Rightarrow MN \parallel AD'$ și $MN = \frac{AD'}{2}$

QP l mijl $\triangle DD'A \Rightarrow QP \parallel AD'$ și $QP = \frac{AD'}{2}$ $\Rightarrow$

$\Rightarrow MN \parallel QP$ și $\boxed{MN = QP}$

b) M, N mijloace $\Rightarrow MP \parallel BC, MP \equiv BC \Rightarrow MBCP$ paralelogram $\Rightarrow MB \parallel PC$

Cum $MN \parallel PQ$

$MB, MN \subset (MNB)$ și $MB \cap MN = \{M\}$

$PC, PQ \subset (QPC)$ și $PC \cap PQ = \{P\}$

$\Rightarrow (MNP) \parallel (QPC)$

$\Rightarrow (QPC) \parallel (MNP)$

Dar, $P, Q \in (QPC)$

$\Rightarrow PQ \subset (QPC)$ și $T \in PQ \Rightarrow$

$\Rightarrow T \in (QPC)$

$C \in (QPC)$ $\Rightarrow CT \in (QPC)$

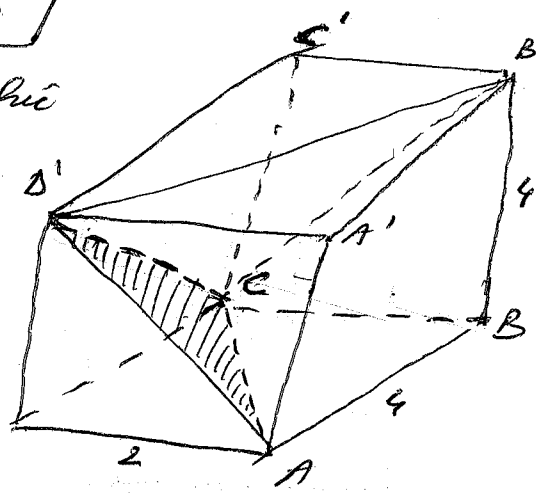
$\Rightarrow CT \parallel (MNP)$

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$ABCDA'B'C'D'$ paralelipiped dreptunghic cu $AB = AA' = 4$ cm, $BC = 2$ cm

a) arată că $A_t = 64$ cm²

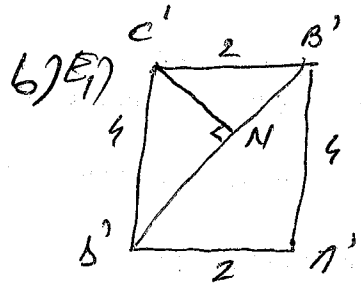
b) dacă N e proiecția lui C' pe $B'D'$ și P proiecția lui C' pe CB', arată că $NP \parallel (ACD')$



a) $A_t = 2(L \cdot l + L \cdot h + l \cdot h) = 2(4 \cdot 2 + 4 \cdot 4 + 2 \cdot 4)$ cm² =

$= 2(8 + 16 + 8)$ cm² $= 64$ cm²

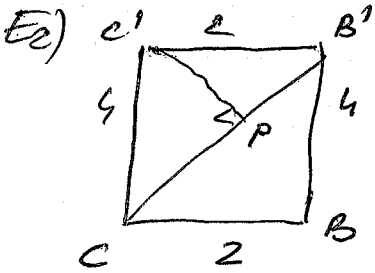
b) E1)



$A'B'C'D'$ dreptunghi, $B'D'^2 = B'C'^2 + C'D'^2$

$\Rightarrow B'D'^2 = 4 + 16 = 20 \Rightarrow \boxed{B'D' = 2\sqrt{5}}$

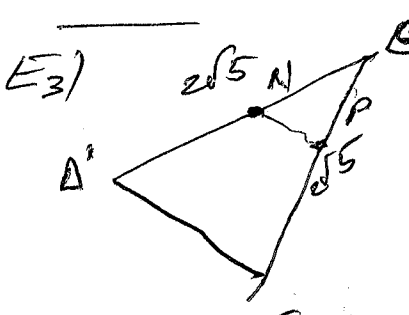
E2)



Analog, $B'C'CB$ dreptunghi $\Rightarrow B'C^2 = B'C'^2 + C'C^2$

$\Rightarrow B'C^2 = 4 + 16 = 20 \Rightarrow \boxed{B'C = 2\sqrt{5}}$

E3)



Studiez dacă $NP \parallel D'C$ (?)

Revenim la E1), T. catetei $\Rightarrow C'B'^2 = D'B' \cdot NB'$

$\Rightarrow 4 = 2\sqrt{5} \cdot NB' \Rightarrow NB' = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Analog, la E2), T. catetei $C'B'^2 = CB' \cdot PB'$

$\Rightarrow 4 = 2\sqrt{5} \cdot PB' \Rightarrow PB' = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\Rightarrow \frac{B'N}{B'D'} = \frac{\frac{2\sqrt{5}}{5}}{2\sqrt{5}} = \frac{1}{5}$ și $\frac{B'P}{B'C} = \frac{\frac{2\sqrt{5}}{5}}{2\sqrt{5}} = \frac{1}{5}$

$\Rightarrow \frac{B'N}{B'D'} = \frac{B'P}{B'C} = \frac{1}{5} \Rightarrow NP \parallel D'C$ $\Rightarrow NP \parallel (ACD')$

Dar, $D'C \subset (ACD')$